DOCUMENT RESUME

ED 328 087 FL 019 056

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TITLE Comprehension and Communicative Evidence in Operative Mathematics Instruction.
PUB DATE Apr 90
NOTE 17p.; Paper presented at the Meeting of the World Congress of Applied Linguistics (International Association of Applied Linguistics) (9th, Thessaloniki, Greece, April 15-21, 1990).
PUB TYPE Reports - Evaluative/Feasibility (142) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Classroom Communication; *Concept Formation; Epistemology; Foreign Countries; *Learning Processes; *Mathematical Concepts; *Mathematics Instruction

ABSTRACT

Most students leave school with good or acceptable grades in mathematics, despite the fact that very few actually grasp basic mathematical concepts. Schools appear to be providing ways of acquiring the necessary knowledge without promoting true mathematical comprehension. This is made possible by reducing mathematics, cognitively, to a merely operative level and making unnecessary the integration of conceptualizations into a solution process. Students monitor, interpret, and evaluate the teacher's verbal and non-verbal behavior for recognizable patterns. The structures of this interactive process, and of the subject matter, allow students to limit their solutions to the options made available to them, and manage to produce objectively adequate but not creative solutions. These minimally productive patterns of classroom interaction should be further studied. (MSE)

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Rainer von Kügelgen

COMPREHENSION AND COMMUNICATIVE EVIDENCE IN OPERATIVE MATHEMATICS INSTRUCTION

I am concerned with the following contradiction: on the one hand, very few school students ever really grasp the basic concepts of mathematics; on the other hand, most students leave school with good or poor grades in mathematics. Apparently, school as an institution provides students with ways of producing the knowledge required, without actually understanding the subject matter.

(1)
Calling this a "masquerade" would certainly be an overly simple explanation of the issue. The incomprehension of mathematical concepts has been the subject of extensive pedagogical and didactic research. In the present context, I will touch briefly on just one of the relevant causes, since I want to focus on the apparent paradox between incomprehension and the production of institutionally valid knowledge.The incomprehension of mathematical concepts must be regarded as a consequence of the system of knowledge acquisition which prevails in our schools. More specifically, it is a product of the typical pattern of interaction found in the classroom context: the lingual action-pattern task-setting / task-solving (Ehlich & Rehbein 1985[1]). This pattern is the main form in which accelerated knowledge acquisition is accomplished in school. In very general terms, the task-solving pattern can be described as follows[2]: The teacher divides the subject matter at hand into segments according to his specific plan. The students are involved interactively to work out these segments in developing solution proposals to the tasks by the teacher. However, the relation between the individual segments exists for them only in terms of a temporal succession, i. e. it is additive. The result is a fragmentization and decontextualization of knowledge, which is typical of the educational instruction process. In this sense, the task-solving pattern is particularly inadequate for mathematical subject matter[3]. Students may arrive at a quite reasonable interpretation of a piece of poetry, even if their background knowledge and their subject skills are incomplete. On the other hand, they will not normally come up with the solution to a cosine equation without sufficient knowledge of fundamental arithmetic.

(2)
The production of correct knowledge by school students, which prevails alongside the manifestation of incomprehension, is rarely based on a mathematically logical derivation of concepts. In most cases, it is based on mechanisms which I will refer to as the "**functionalization of communicative evidence**". With the aid of these mecha-

[1] v. the diagram in the appendix, p. 15

[2] cf. Ehlich & Rehbein 1986, Muster und Institution, esp. § 2 and § 7.

[3] cf. v Kügelgen 1989, Interactional problem-solving in mathematic-lessons -an transcript-analysis-

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nisms, students manage to produce objectively adequate, i. e. institutionally valid and viable answers in a manner which is not suited to the subject matter itself.

- Copying, hinting, faking and cheating – in their innumerable guises and forms of creative variation – represent a powerful set of tools which students employ for this purpose. The existence and use of these "traditional" tools is recognized by all parties concerned, although the issue is fiercely battled in institutional practice. Yet, the following types of communicative evidence normally go unnoticed, despite their effectiveness.
- For instance, students can exploit the evidence provided by "blue prints" of the text, the subject matter, and the lesson plan, etc., in order to contribute propositional elements included topically or structurally in these sources at relevant points in the discussion of a topic, without actually having understood the interrelations of the derivation process.
- students can make conclusions about the direction and substance of derivation processes by interpreting the teacher's verbal and non-verbal behavior.
- students can process the questions of the day by generating *operative solution-schemes* according to ritualized procedures without penetrating the subject matter in any mathematically relevant way.
- students can develop highly probable hypotheses about correct solutions – depending on the number of options available in a given context – by means of linguistic and process-logical inference and association procedures and can carry out seemingly mathematical operations based on these hypotheses with no recourse to mathematical knowledge.

Thus, by monitoring and interpreting the teachers evaluations, which are inevitable in the context of the pattern, and by relating these evaluations to the subject matter at hand, the students can systematically narrow down the set of possible solution proposals. In general terms, students can bring their *pattern knowledge* into play in the form of numerous tactics, strategies and techniques, in order to stand their ground successfully in the school environment and produce institutionally valid knowledge despite incomprehension of the actual subject matter.

The development and implementation of these mechanisms is an integral part of classroom interaction; this is true of mathematics as well as any other subject. As such, the institutionally valid knowledge produced in this way turns out to be a kind of *façade knowledge*[4], i. e. it is merely a linguistic costume for the body of knowledge structures relating to the subject.

(3)
Mathematics instruction in school is largely limited to operative action; i. e. due to a specific division of labor, action by students is reduced to the performance of algebraic or numeric operations, whereby any contextualization with conceptual levels of representation is excluded. I will refer to this situation as "**operative instruction**".

Supplemental operation (fill-in-the-blank operation) is the prevailing method of knowledge processing in operative instruction. By that I mean that the actual achieve-

[4] I am indebted to W. Grießhaber for this terminology.

ment of transforming the subject matter and its interrelations into mathematical forms of representation is already provided, e.g. in the form of hints contained in the formulation of a task itself or in the task-solving process. In particular, the necessity of conceptualization is suppressed: objects and entities are already expressed in numbers and the relations between them are already converted into operations, so that students are merely required to carry out the operative steps and links introduced by the teacher.

The type of knowledge and action demanded of students in operative instruction can be generated by means of the mechanisms described here more economically than by means of mathematical understanding. However, the knowledge generated by the functionalization of communicative evidence is merely façade knowledge, which includes no understanding of mathematics, but is sufficient to fulfill institutional requirements.

In fact, the acquisition of mathematical understanding is normally contraposed to the achievement of learning objectives, since it would require the revision and elimination of deficits built up and accommodated over years of instruction.

(4)
Deficits of this kind are revealed in the segment of classroom discourse analyzed below: Despite the *communicative pressure* resulting from embarrassment and discredit, one of the students refuses to accept the communicative evidence provided by the correct solution to an exercise in division as a replacement for mathematical knowledge. The lack of mathematical understanding first appears in the form of an open manifestation of incomprehension. This manifestation of incomprehension is itself an anomaly, which is contrary to the natural flow of classroom interaction-structures. This leads to an unplanned discussion on the concept of mathematical division. The teacher makes various attempts to repair the expressed incomprehension. However, these attempts are not focussed on the fundamental conceptualization of mathematical principles. On the contrary, they consist in generating an operative scheme behind which the conceptual deficits continue to exist.

At the same time, an *interactive routine* is generated by the students' drilling of the operative scheme. With respect to its conceptual substance, the routine does not even entail mastering performance of the algebraic-numerical operations correctly; rather, it reduces them to a kind of *conditioning* in the form of a linguistic convention.

As a result of this operational approach, the use of communicative evidence to produce institutionally valid knowledge -more specifically: façade knowledge- is confirmed, while any mathematically relevant re-processing of the deficit knowledge is removed even further into the background of attention.

Thus, an analysis of this episode will document the very process which ends in the contradiction cited here by way of introduction.

(5)
These hypotheses will be illustrated below by the interpretation of a segment of classroom discourse (v. the appended transcription, pp 12):

At the beginning of the sequence, a knowledge deficit is branded as a substantial failure ((s176) *"What exactly is b divided by one?"*)[5] and considerable communicative pressure is generated with a view to eliminating this deficit. The element of knowledge at hand is not made accessible by re-processing the underlying deficit, but by communicative evidence (assistance from a competent student (s195)). Up to this point, the events do not depart from the institutional routine of generating knowledge by means of tactical and strategic pattern-knowledge implementation. Subsequently, a very unusual phenomenon occurs, in which the routine procedure is *reversed*. Rather than being glossed over, incomprehension is unveiled by a refusal to accept the use of pattern knowledge, thus providing an opportunity to deal with the deficit in a problem-solving process.

(6)
In the subsequent discussion of the question "What exactly is b divided by one?", the teacher (T) generates communicative pressure right from the start, in order to eliminate the knowledge gap as quickly as possible. Paradoxically, this objective is inverted to its direct opposite[6], viz. the consolidation of incomprehension by the use of communicative evidence and the drilling of operative solution-schemes.

The initial characterization of Mustafa as an
(s179) T: *expert on third-grade arithmetic*
is a debasement which, because of its ironic nature, cannot simply be dismissed. By employing the weapons of irony, the teacher functionalizes his linguistic superiority. Encountering this strike on the same level requires considerable sophistication, routine and fine tuning of the mental apparatus, as well as the ability to employ corresponding linguistic resources. This represents a threshold not only for foreign students. If the response given is too blunt, the student will appear to be in the wrong; if it is too weak, it will tend to indicate consent. The effect of aiming irony at students (or children in general), who are both mentally and linguistically inferior to the teacher in this respect, is one of intimidation: the students are lead to believe that their insecurity is not a result of outside factors, but of their own incompetence.

The teacher's use of irony in this specifically indirect manner allows a certain aggression to be released, which stems from his helplessness in dealing with the detected knowledge deficit. The students' inability to perform operations such as "b divided by one" or "a divided by a" and the outrageousness of some of the solution proposals express a schematic concept of elementary arithmetic founded in the operative instruction of primary-school mathematics. In terms of this concept, anything that goes

[5] The small numbers in parentheses count the utterances of the transcript; citations stand in *italic*.

[6] cf. v. Kügelgen, 1990: "Aufgaben- und problemlösendes Lernen. Musteranalytische Untersuchungen im Mathematikunterricht bei jugendlichen Ausländern" (dissertation, to appear), § 1.10.2. "Das Paradox des Aufgabenlösens".

beyond the performance of supplemental operations stands on shaky ground.

The teacher is well aware that deconstructing this schematic concept of the "divided by" operation, which he tries to achieve to a certain extent in the course of the discussion, is a futile task. However, he compensates his frustration over the irreparability of the knowledge deficits by passing it on to the students. As a result, the deficits are suppressed, rather than being confronted. In this respect, the other students' laughter not only gives expression to the successful use of irony and the demaskation of Mustafa, but also to the feeling of relief that someone else got it this time.

But T does not want to let the event pass:

(s182f) T: *What is b divided by one?* ((to Fahrudin)) *Come on, **you** tell us!*

This forced turn allocation under severe conditions

(s184) L: ***Be quiet! Nobody** say anything! b divided by one, come on!*

teaches Fahrudin a social lesson about laughing at others when he doesn't know the answer himself. Fahrudin tries to get out of the line of fire by re-interpreting his reaction as not being a "laughing at" type of laugh:

(s187) Fahrudin: *No, I only wanted to...* ((-laughs-))

but he can't think of a credible explanation; finally, he tries to appease the situation by laughing, which in this case is meant to be understood as saying that he was not laughing about Mustafa's demaskation but about an imaginary joke by the teacher independent of its victimizing effect. However, L is not prepared to transfer the entire event to the playful level; instead, he maintains the original illocutionary quality of Fahrudin's action:

(s188) L: *You laughed at him!*

In other words: don't laugh at someone else, unless you know better yourself. The other students also express their awareness of Fahrudin's diversion tactic:

(s189) Harun: *Because, he doesn't know!*

(s190) Ataschin: *He doesn't know!*

As such, Fahrudin's tactic is no longer supported by anyone else. He decides to present a gesture of humility, although by now Mustafa has proposed a possible solution to the question following non-verbal agreement with his neighbors Birol and Behiye, and although it is apparent to Fahrudin that this solution proposal is very probably correct:

(s191) T: *Well then?*

(s192) Mustafa: ***b!***

However, Fahrudin also knows by experience that in critical situations like this, L is certainly capable of carrying on this intimidation tactic or even intensifying it by requiring exact explanations for adopted solutions. For this reason, he decides to play dead:

(s193) T: *And what do you have to say?*

(s194) Fahrudin: *Nothin'.*

The teacher's frustration over the futility of actually eliminating such deficits in mathematical knowledge adequately is compensated by social and communicative gestures of humility from those affected by the deficiency. Experiences of this kind are reflected in the school achievement principle in rules like

- *If you get caught, it's your own fault!*

- If you don't know what you're talking about, you ought to keep your mouth shut!
- If your achievements are good, your conduct doesn't need to be great!

Faced with the dangers of image loss and negative evaluations associated with the victimization tactic, the students will resort to any suitable means in order to conceal prevailing deficits. At the same time, no assistance is provided in dealing with the deficits appropriately, so that generating communicative pressure in this way leads students to fall back increasingly upon the ersatz-mechanisms of communicative evidence in producing institutional knowledge, i. e. it results in a stabilization of operative instruction.

(7)
The following events must be evaluated in this framework: despite the use of considerable communicative pressure by the teacher, Kevin refuses to adopt an element of knowledge provided in the form of pattern-relevant evidence and to produce institutionally valid façade knowledge.

The concepts of division – or of algebraic operations in general – prevailing on the part of the students are revealed in this context. Integrating these concepts into the problem-solving process might have led the discussion directly to the heart of the students' conceptualization of mathematical principles. However, instead of seizing this opportunity, the teacher conceals the apparent mathematical-logical voids by stabilizing the operative scheme. This example indicates how pattern and subject knowledge, with their different knowledge qualities, can be transformed and interchanged in a positive or in a negative sense.

In any case, the correct solution is clearly provided by Mustafa's:
(s192): ***b!***
and Behiye's confirmation:
(s195): ((to Mustafa)) *Right!*
So that, at this point, Kevin's open disclosure of not knowing the answer
(s200): ***I don't know!***
clearly departs from the task-solving pattern. In Kevin's terms, "knowing" refers to a concept of true comprehension, not just passing the buck by providing pattern-evident solution proposals[7]:
(s202): *Hey, at least I'm being honest!*
Kevin apparently feels that concealing his own incomprehension would be "dishonest", and his reluctance to do so leads him to turn down the invitation to adopt the answer resulting from the application of pattern-evident assistance. Initially, T tends to regard Kevin's statement of incomprehension as a joke and asks Kevin to confirm the illocutionary quality of his utterance:
(s203) L: *b divided by one?*
Kevin is not about to give up his position: In his next utterance, he goes on to deal with the subject matter by offering an alternative to the solution proposed and con-

[7] cf. Ehlich & Rehbein, 1986, "Muster und Institution. Untersuchungen zur schulischen Kommunikation" § 2.4.

firmed in the flow of communicative evidence in the preceding sequence:

(s207): *Could also be one.*

By doing so, Kevin runs the risk of a severe negative evaluation, because the issue applies to fundamental mathematical knowledge.

(s208) L: ((toneless)) *Could also be one.*

In the face of this revelation of operative arbitrariness, L is utterly perplexed. Kevin does much of the work for him by explicating the contradictions included in his repertoire of methods and principles:

(s209-217) Kevin: ***Yeah, look!*** *I don't* ***know****. One divided by b is one, ...b divided by one is b.*

Here, Kevin is presumably carrying over the principle of logical inversion from the context of common sense conceptions to the level of algebraic-symbolic operations: He seems to think that, if "divided by one" causes no change, then its inversion, "one divided by ... " must have the same effect, i. e. no change. In this sense, the assertion that Kevin makes is logical: *"One divided by b is one, ... b divided by one is b"*.

According to this system of logic, the "divided by" operation is a procedure such as *is linked with*, which remains unchanged in its reciprocal. Kevin apparently comes to this conclusion by generalizing the equation "b divided by one equals b".

Once the distinction between the two operations is obscured, Kevin comes to the conclusion – paradoxically – that "one divided by b" and "b divided by one" represent exactly the same question, and that the correct answers can be exchanged at will: "b" or "could also be one".

In the light of these considerations, the ambiguity of Kevin's approach is apparent: he transfers common-sense procedures to mathematics summarily, without differentiating them sufficiently; and he evaluates algebraic operations according to common-sense rules, without understanding their specific predicative qualities.

Yirgalem does not want to get entangled in Kevin's train of thought, but he does pick up on the issue in a more general sense by questioning whether it is permissible to correlate numeric and algebraic-symbolic categories in this way:

(s226) Yirgalem: *One, can you divide ... ... letter by numbers?*

Kevin responds to this utterance directly, denying that the type of operation Yirgalem is referring to is possible:

(s229) Kevin: *Nope, you can't! ..Apple divided by ten marks.... you can't do that!*

Thus, he applies the concept of the DIVISION/SUBDIVISION variant of the division operation (*"b divided by one"* or *"can you divide letters by numbers?"*) to the "DISTRIBUTION/ALLOCATION variant ("apples divided by ten marks") of this operation. He is concerned with what exactly a quotient of different entities represents, i. e. whether dividing units from different ontological categories is permissible.

Obviously, dealing with the issues at hand requires investing a certain effort in the conceptual basis of the division operation. Individual students have expressed not only aphoria, but also an interest in learning and understanding what they do not know,

and they have formulated their conceptions of the task and its possible solutions: by their rejection of communicative pattern evidence an entrance in problem solving in school[8].has been called for and also become possible. However, the teacher simply ridicules and disqualifies the entire issue by classifying it as

(s237) T: *third-grade math.*

The other students readily pick up this signal in their reactions of laughter and amusement, e. g.

(s219 - 223) Michael: *Oh no! Oh God, no!*

(s259) Sn: ((---increasing amusement ---))

To a certain extent, L continues to put communicative pressure on Kevin and Yirgalem via the students, who pass on his communicative signals. By doing so he forces them to refrain from focussing on problem-solving investigation of the subject matter, he expects them to apply the façade knowledge evident in the operative scheme, and to adapt the subsequent explanation of the problem:

(s242ff) T: *One divided by one, Kevin? ... Two divided by one? ... Three divided by one? ... Three point seven zero divided by one?*

T employs the evidence of the tasks to defame Kevin's and Yirgalem's fundamental questions as being utterly elementary. His approach consists of moulding an operative scheme by posing a series of analog questions based on the maxims: "keep it simple!" and "proceed from the known to the unknown!" The conceptual difficulties are inveiled in operative interaction. The rule for producing the linguistic surface structures for the required knowledge is as immediately evident as it is insignificant:

(s249) Michael: ((laughs)) *It's the same thing every time.*

In the task-solving process comprehension is replaced by pragmatic viability, initiated by conditioning and stored in the form of solution schemes.

The question

(s265) T: *Apple divided by one?*

is intended to extend the scheme in the direction of the task at hand "What exactly is b divided by one." Michael mocks the simplicity of the answer in

(s267): ((jokingly)) *Orange.*

However, Kevin once again rejects the use of the scheme. Instead of concluding the episode successfully, e. g. with *"*yeah, same thing every time: apple"*, he reiterates the difficulties expressed previously:

(s266, 269) Kevin: *Yeah, I don't know. You can't/*

At this point, T interrupts Kevin's utterance, so that no new errors are produced before the difficulties at hand have been eliminated: T is not really interested in what is going on in Kevin's head. Rather, he is set on repairing interferences in the execution of his plan.

The teacher's move is not an act of RECEPTION, corresponding to position 7 in the instructional problem-solving pattern[9], i. e. the reconstruction of the student's mental

[8] cf. Kügelgen, op.cit., § 1. esp. § 1.5. – § 1.10.

[9] v. the diagram in the appendix, p. 16 ; cf. Kügelgen, op.cit., esp. § 1.9. – § 1.11.

process; rather it is an EVALUATION, corresponding to position 10 (or 16) in the task-solving pattern, i. e. the process of fitting the fragments of knowledge offered by students into a pre-defined plan or of rejecting the knowledge elements. In this context, either the correct answer to the question "Apple divided by one?" is required – or no answer at all.

Kevin's rejection of the operational scheme forces the teacher to give a more extensive explanation of the subject matter:

(s270-299) L: *Okay, what does "divided by one" really mean? That's the question. We need to figure out what "divided by one" means. How many apples do you have? ... And how many people? ... So how many people are the apples distributed to? ... How many does he get? ... So, what is apple divided by one? ... One apple. It's one apple.*

Yet, the very source of the difficulties lay in Kevin's and Yirgalem's thinking aloud about "What 'divided by one' really means". The concept of DISTRIBUTION, which L explains, is no help to either Kevin's or Yirgalem's problems. The original task, "b divided by one", is not a question of DISTRIBUTION or ALLOCATION, but of DIVISION or SUB-DIVISION.

Yirgalem's problem lies in an abstraction of the division concept: He is concerned with whether it is permissible to extend the concept of SUB-DIVISION to the level of algebraic symbols (*"can you divide letters by numbers?"* (s226)). He is engaged in contextualizing the operation from the level of a general, common-sense understanding of mathematics to the level of algebraic-symbolic representation.

In Kevin's example (*"Apple divided by ten marks, ... you can't do that"* (s232), Yirgalem's approach to abstracting the concept of DIVISION/SUB-DIVISION is incorrectly paraphrased in terms of the concept of DISTRIBUTION/ALLOCATION (*"Apple divided by ten marks, ..."*). Once the concepts have been equated, Kevin is right in declaring that the proposed division is not permissible (*"... you can't do that"*). As a result of this equation of concepts, Kevin once again denies the divisibility of entities because of their ontological invariability: (T: *Apple divided by one?* Kevin: *Yeah, I don't know. You can't!* (s256ff)).

I will refer to this diffuse intermixing of different concepts and the mutual suppression of their respective logical-predictive qualities as **"confluence"**.

T's explanations miss the point with respect to the student's incomprehension of the matter, as the following paraphrase indicates:

* "If one apple is distributed among one person, then the person gets one apple. *Hence*, apple divided by one is apple."

Clearly, the second part of the explanation ('Subdivision of an entity') is not a logical consequence of the first ('Distribution of an entity to one or more other entities'). On the contrary, a confluence is generated between the concepts of DISTRIBUTION/ALLOCATION on the one hand and DIVISION/SUB-DIVISION on the other which often returns to the teacher as its originator, e.g. in the incorrect reduction of units.

Confluence is a phenomenon which occurs frequently in the classroom context. In this case, it is based on an unawareness of the complex and contradictory nature of the mental concepts relevant to the given mathematical operation. Extensive experience allows the "specialist" to filter out these contradictions automatically and unconsciously when performing the operations. Without the filtering effect produced by this base of experience, the contradictions are called up whenever performance requires comprehension, i. e. conceptualization.

In other words, T displays the same confluence as Kevin. For this reason, he cannot access the cognitive essence of Kevin's incomprehension. Nevertheless, he is able to operate correctly, since his experience and routine enable him to eliminate false or contradictory alternatives unconsciously. Under these circumstances, the extent of his explanation is naturally limited to a description of the operation. Comparable circumstances normally prevail in task-solving, because – according to the pattern – the solution proposals by the students can only be evaluated with respect to whether they fit properly into the completion of the teacher's plan. A deficit analysis by the teacher which reaches the crucial cognitive point requires prior RECEPTION of the student's solution proposals according to position 7 of the pattern of instructional problem solving. Without this RECEPTION, attempts at explaining the subject matter merely result in the production of a functional operative scheme ('experience-knowledge'). Reducing the cognitive focus to operative schemes is characteristic of task-solving pattern in math and science instruction.

Within this framework, attaining true progress on the cognitive level is essentially contraposed to the implementation of the structures of the task-solving pattern. For Kevin, this progress consists in realizing that when an entity is divided/sub-divided, it's ontological category remains unchanged:

(s301 - 303) Ke: *And sugar divided by ten?* T: *What's that?* Ke:*Yeah, sugar.*

He works this out for himself contrary to the teacher's focus of attention which is centered on the *quantity* resulting from the division process:

(s304) T: *Yeah, and how much sugar?*

and, in particular, in opposition to the teacher's confluence of the DISTRIBUTION/-SUB-DIVISION concept and the DISTRIBUTION/ALLOCATION concept.

Even with respect to the teacher's focus on quantification it is Yirgalem who evaluates the mathematical essence of the divided-by-one operation conceptually in

(s304) *He gets the whole thing.*

whereas T ignores this decisive process of conceptualization by limiting the operation to the numerical level via his analogy to the apple example.

(8)

The episode discussed here documents the unfortunate role of operative instruction. Cognitive difficulties encountered on the conceptual level (or in the contextualization of the conceptual and symbolic levels) are suppressed/re-interpreted and concealed by the generation of operative schemes. In the absence of any reception of or consideration for the students' specific cognitive conditions and requirements, knowledge fragments are produced and deficits are repaired only when doing so fits with the com-

pletion of the teacher's plan. For this reason, producing and storing these knowledge fragments inevitably involves the implementation of operative schemes and the mechanisms of communicative evidence. If, in addition, the production of operative schemes is said to supposed an explanation of the subject matter, as in this case, the resulting conceptualization of mathematical principles is limited to pragmatic functionality at best.

The division of labor in the task-solving pattern instruction – design, processing and the providing of clues on the basic propositional content on the teacher's part, operative assistance by the students – has a tendency to be consolidated in operative instruction: the teacher only includes elements of the total problem in his discussion plan and in the specific process of analysis and synthesis[10] expressed in the form of key questions, instruction sequences, etc. - more specifically: only such elements which are directly related to the performance of algebraic and numeric operations and guarantee nothing but their pragmatic implementation.

Of course, the use of communicative evidence in the classroom context is as inevitable as the implementation of pattern knowledge itself. Both types of knowledge are integral parts of institutional communication. The negative effects of these mechanisms with respect to comprehension consists in their functionalization, i. e. their employment in the production of institutionally valid knowledge. In order to keep these negative effects under control, it is important to maintain a distinction between conclusions drawn from the mathematical subject matter itself and conclusions arising from the social-communicative situation, in which the subject matter is processed. By doing so, it is possible to establish the inadequacy of functionalizing communicative evidence as a means of producing subject knowledge.

One consequence of these findings for teaching practice is the necessity to include the methods of knowledge acquisition among the topics of institutional discourse: critical reflections on the functionalization of communicative evidence are prerequisite to a problem-solving approach to knowledge acquisition.

Operative instruction, on the other hand, represents the extension of a learning concept to the realm of science and mathematics which should never have been allowed to escape from the pigeon cages of behaviorist theory.

>>>>>>><<<

APPENDIX

270286/mathematics instruction/von Kügelgen/pre-vocational instruction for foreigners (Realschule), 10th grade/standard procedures in the calculation of triangles/cosine equation/24'50"/2-111187/ von Kügelgen /1:50/Akai GX-F71/Video Panasonic NV-8200/T teacher, At Ataschin, Be Behiye, Bi Birol, Da Daud, Fa Fahrudin, Ha Harun, Hi Hai, Ke Kevin, Kh Khalid, Ma Mahiddin, Mi Michael, Mu Mustafa, Yi Yirgalem, S one, Sn several unidentified students/ !! important location

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	b divided	T 176 *What exactly is b divided by one?*
	by one	Mi 177
47	**-6-**	

T 178 Mustafa? 179 – Expert on third-grade arithmetic – 182 What is b divided by
Mi Gee..... 180 ((———*-laughs-*———))
Be, Fa, Sn 181 ((----------*-loud*
48

T *one?* 183 *((to Fahrudin)) Come on,* ***you*** *tell us!* 184 **Be quiet!** 185 **Nobody** say anything! 186 b
Be, Fa, Sn *laughter -------))*
49

T divided by one, come on! 188 **You** laughed at him! 191 Well then? 193 We're waiting. What do you
Fa 187 No, I only wanted to.../ *((-laughs-))*
Ha 189 'Cause he doesn't know.
At 190 He doesn't know.
Mu 190a You tell us! 192 **bee**
50

T have to say? 196 And what do **you** have to say? 199 And you? *((points to individual students))*
Fa 194 Nothin'.
Be 195 *((to Mustafa))* Right! 201 *((- laughs-))*
Mi 197 I say b.
Mu 198 b.
!! Ke 200 **I** don't **know.??**
51

T 203 b divided by one?
Be 205a **Yeah!**
!! Ke 202 Hey, at least I'm being honest! 207 Could also be
At 205 **b´ee`!**
Da 206 b divided by the sine of
52

T 208 *((toneless))* Could also be one.
!! Ke one. 209 Yeah, look! 211 I don't know. 212 One divided by b is one,
Da ninety degrees.
Be 210 ((*--laughs--*))
Yi 213 As if b were, as if b
53

|T
|Ke .....212a .....b divided by one is b.
|Da 214 Do you want me to? 216 b divided by the sine of beta
|Ma 215 Hey, do you mean b divided by one, or one divided by b?
|At 218 *((-----laughs------))*
|_54

|T 224 Hold on! The question is b divided by one
|Mi 219 Ooh n´oo`! 222 Oh Go`d,... 223 No!
|Ma 221 b divided by the sine of beta.
|_55

|T 230 Yeah, Daud, we know. Yeah, we know. 236
!! |Yi 226 One, can you divide letters... by numbers?
|Da 227 Ja. Yeah, it's one. 228 The angle A/ A-...D-...C... is ninety degrees. 231 b divided by ... is/
!! |Ke 229 Nope. 232 Apple divided by ten marks ...
|Be *((to Daud in angry tone))* 233 What rubbish you are talking!
|At *((enervated))* 234 Yeah.
|_56

|T The question/ 237 Right now we're in the third grade and we're arguing about ... third-grade
|Ke you can't do that!
|Be 235 The question is b divided by one. 238 *((-giggles-))*
|At 239 ((--*giggles*-))
|_57

|T math. 242 One divided by one, Kevin?? 244 **Two** divided by one? 248 **Three**
|At -- 240 ((*Translates "b divided by one" for Daud*))
|Da 241 I already told you.
|Ke 243 One. ((*uncertain*)) 245 Two.
|Mu *((jokingly))* 246 Six.
|Mi *((tentatively))* 247 Two.
|_58

|T divided by one. 251 Three point seven zero divided by one. 255 Divided by one.
|Ke 250 Three 252 Three point seven zero? 256 Are
!! |Mi 249 *((-laughs-))* It's the same thing every time.
|Mu *((hesitant))* 253 Three point seven...
|Ma 254 Are, uhm ...
|_59

|T 260 Well then? 261 Three point seven zero divided by one?
|Ke Zero point .....258 twenty one.
|At, Be *((---laugh---))*
|Sn 259 *((------increasing amusement---------))*
|Mi 262 Three point seven zero.
|_60

|T 264 Yeah. 265 Apple divided by one? 270 Okay,
|Ke 263 Same thing? 266 Yeah, I don't know. 269 You can't/
|Mi *((joking))* 267 Orange.
|At 268 *((--laughs--))*
|_61

|T what does "divided by one" really mean? 271 That's the question. 272 We need to figure out what
|_62

|T "divided by one" means: 273 How many apples do you have? 275 And how many people? 279
|Ke 274 One. 276 One.
|Mi 277 One.
|Hi 278 One.
|_63

!! |T So, how many people are the apples distributed to? 284 How many does he get? 288
Now,
|Ke 280 One. 286 One
|Mi 281 One.
|Hi 282 One.
|At 283 Just one **per**son. 285 One.
!! |Yi 287 He gets
|_64

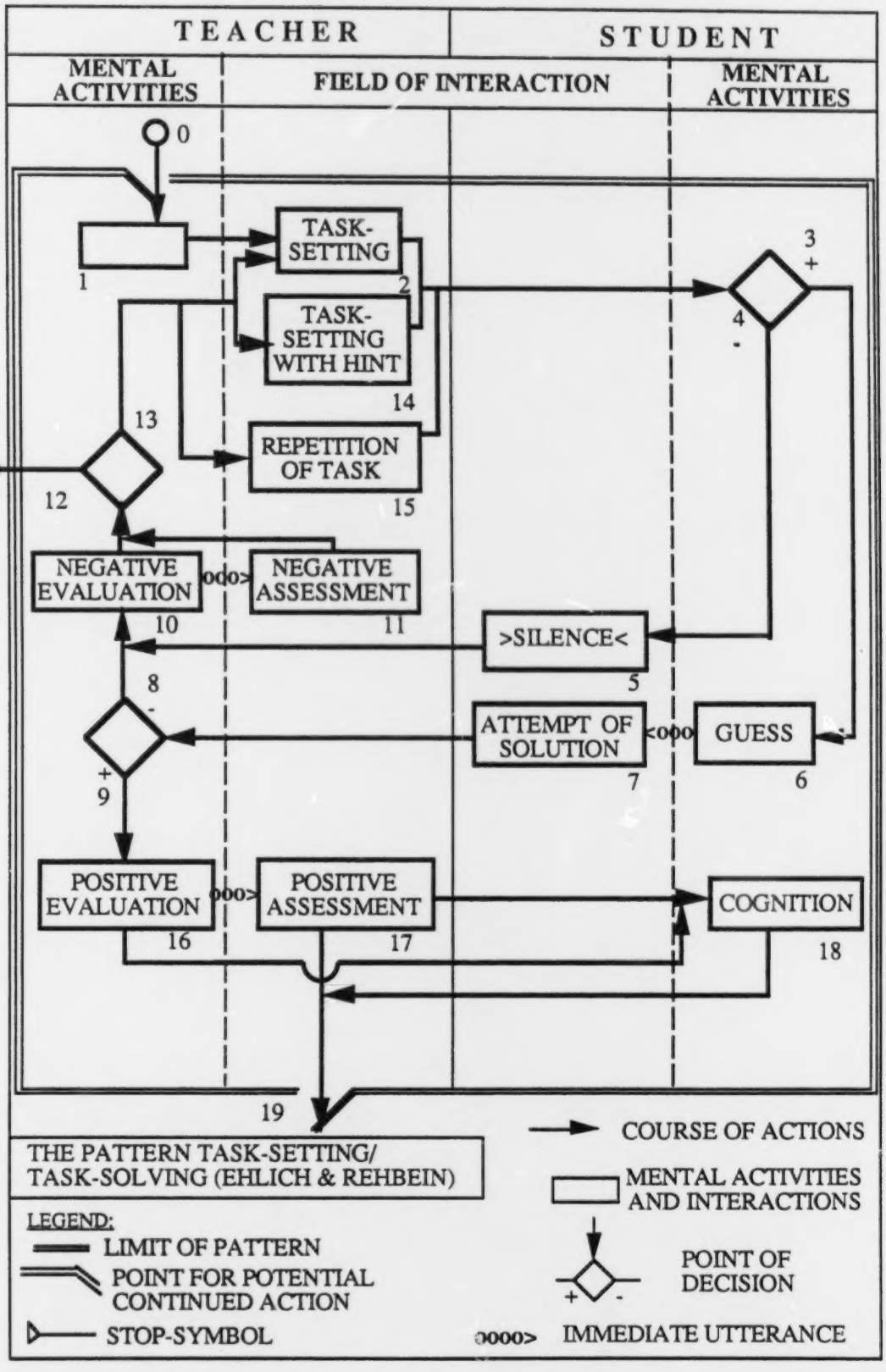
|T **what** does he get "one" of? 291 One **apple**. 292 So, what is apple divided by one? 297
|Yi the whole thing. 295 Apple.
|At 289 One apple. 293 Apple.
|Mu 290 One apple. 296 Apple.
|Ke 294 Apple.
|_65

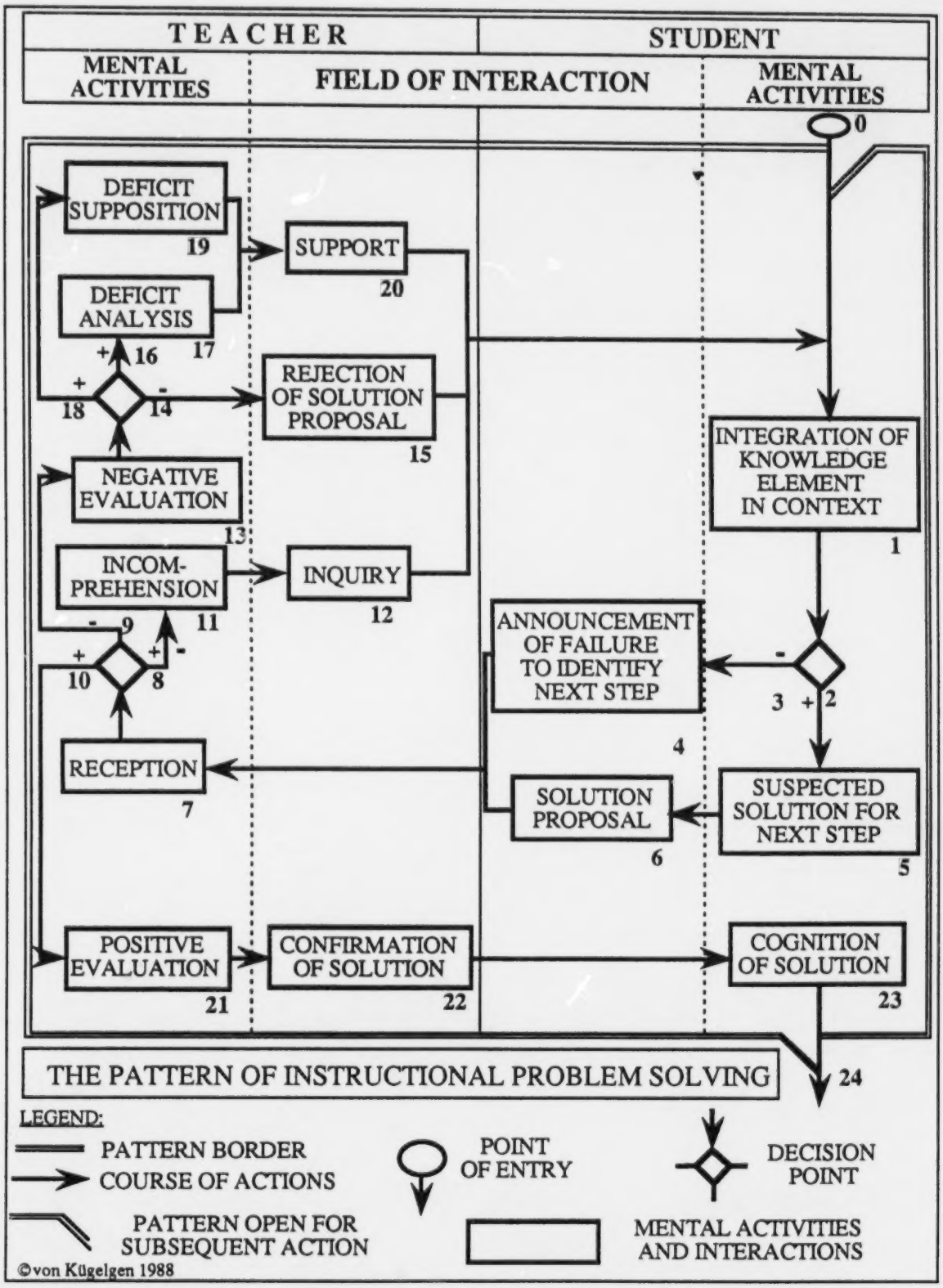
|T One apple. 299 It's one apple. 302 What is that? 304
|At 298 Yeah.
|Be 300 I thought it was raspberry jam! *((-laughs-))*
!! |Ke 301 What about sugar divided by ten? 303 Uh, sugar
|_66

|T Yeah, and how much sugar? 308 Approximately? 309 One tenth? 310 It's one tenth of the
|Mi 305 You can measure it.
|Ke 306 Uh, ten.
|Hi 307 One tenth.
|_67

|T sugar. 311 Right? 312 And what is apple divided by ten? 314 One tenth of an
|Ke, Hi 313 One tenth of an apple.
|_68

|T apple, right? 315 That's it. 316 Okay. 317 Now, what is b´e` divided by one? 319 bee. 320 Okay.
|Be, Mi, Ke 318 be :
|_69


TEACHER
STUDENT
MENTAL ACTIVITIES
FIELD OF INTERACTION
MENTAL ACTIVITIES
0
1
TASK-SETTING
2
TASK-SETTING WITH HINT
14
REPETITION OF TASK
15
3
+
4
-
13
12
NEGATIVE EVALUATION
10
NEGATIVE ASSESSMENT
11
>SILENCE<
5
8
-
+
9
ATTEMPT OF SOLUTION
7
GUESS
6
POSITIVE EVALUATION
16
POSITIVE ASSESSMENT
17
COGNITION
18
19
THE PATTERN TASK-SETTING/ TASK-SOLVING (EHLICH & REHBEIN)
COURSE OF ACTIONS
MENTAL ACTIVITIES AND INTERACTIONS
POINT OF DECISION
LEGEND:
LIMIT OF PATTERN
POINT FOR POTENTIAL CONTINUED ACTION
STOP-SYMBOL
IMMEDIATE UTTERANCE



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